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NONSTATIONARY INTERACTION MODE IN A FLUCTUATING BOUNDARY LAYER

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The limit properties of the Cauchy problem for a partial integrodifferential equation describing the nonstationary interaction in a fluctuating boundary layer are examined for large Reynolds numbers Re [1]. It is shown that if the minimal value of the surface friction stress per period in front of the interaction domain is negative and greater in order of magnitude than $\mathrm{Re}^{-1/8}$, then a range of wave numbers are extracted in the perturbed solution spectrum for which excitation of appropriate harmonics occurs in a bounded time interval. The physical mechanism of the excitation is discussed as an appearance of an instantaneous flow instability. A classification is presented of the limit flow modes in the case when the minimum of the unperturbed friction is positive and much greater than $\mathrm{Re}^{-1/8}$.

1. INTRODUCTION

The flow with interaction in a fluctuating boundary layer is studied in [1] in an incompressible fluid around a flat plate with a small local surface deformation. An asymptotic theory of the flow for large Re was constructed under the assumption that the stream outside the boundary layer does not change direction during the whole time period while the minimum of the unperturbed friction stress on the plate in front of the domain of interaction is a quantity equal to $O(\mathrm{Re}^{-1/8})$ in order of magnitude. It turns out that an investigation of three characteristic interaction modes is required to a lesser degree, in order to construct a solution uniformly suitable in time. Most interesting is the nonstationary interaction realized in a time interval of duration $O(\mathrm{Re}^{-1/16})$ when the unperturbed friction on the plate is almost a minimum. The flow in this interval is described by the Cauchy problem formulated in [1] for the nonlinear partial integrodifferential equation

$$\frac{\partial B(X, T)}{\partial T} = -\gamma \frac{\partial}{\partial X} \int_{-\infty}^{X} \Big\{ [B(\xi, T) + f(\xi)] [T^2 + \sigma + H_0 (B(\xi, T) +$$

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$$+ f(\xi))] - \frac{1}{\gamma_0 2^{5/4}} \int_{\xi}^{+\infty} \frac{\partial^2 B(s, T)}{\partial s^2} \frac{ds}{(s-\xi)^{1/2}}\Bigg\} \frac{d\xi}{(X-\xi)^{3/4}} \tag{1.1}$$

$$B(X, -\infty) = -f(X), \; B(-\infty, T) = f(-\infty) = 0.$$

Here T is the reduced time, X is the longitudinal coordinate, γ and γ_0 are fixed positive constants, H_0 is the effective amplitude of plate deformation and the function f(X) governs the deformation mode. The desired function B(X, T) is simultaneously the perturbation of the longitudinal velocity component and the surface friction stress as well as the thickness, taken with opposite sign, of the near-wall viscous layer displacement thickness. The combination $T^2 + \sigma$ yields the unperturbed friction stress on a plate whose minimum is proportional to σ and is achieved at the time T = 0.

The correctness of the formulation of the problem (1.1) was studied in a linear approximation $H_0 = 0$ [1]. It turns out that the solution exists, depends continuously on the deformation mode and as $T \to +\infty$ approaches the quasistationary limit $B(X, +\infty) = -f(X)$ asymptotically. The solution is stable relative to perturbations produced at an arbitrary time if the induced perturbation has a bounded spectrum. However, in the case of perturbations with an unbounded spectrum the problem on stability turns out to be incorrectly formulated in the general case, which is related to the explosive growth of the amplitude of the shortwave modes in the initial stage of evolution of the solution.

2. EXCITATION IN A BOUNDARY LAYER WITH STRONG COUNTERFLOWS

Let us examine the case of infinitesimal deformation to which a linearized form of the problem (1.1) (H_0 = 0) corresponds. The initial equation is simplified by using the Fourier transform

$$\frac{\partial B^*}{\partial T} + \left[\gamma_0 (i\omega)^{3/4} (T^2 + \sigma) + \gamma_1 (i\omega)^{5/4} |\omega|\right] B^* =$$
$$= -\gamma_0 (i\omega)^{3/4} (T^2 + \sigma) f^*(\omega), \quad \gamma_1 = 2^{-5/4}\pi^{1/2}, \tag{2.1}$$
$$B^*(\omega, T) = \int_{-\infty}^{\infty} \exp(-i\omega X) B(X, T)\, dX, \quad |\arg(i\omega)| < \pi.$$

The solution of this equation that satisfies the initial condition $B^*(\omega, -\infty) = -f^*(\omega)$ has the form

$$B^* = -f^*(\omega)\Bigg[1 - \gamma_1 (i\omega)^{5/4} |\omega| \int_{-\infty}^{T} \exp\Big\{\frac{\gamma_0}{3}(i\omega)^{3/4}(s^3 - T^3) + \tag{2.2}$$
$$+ \left[\gamma_0 \sigma (i\omega)^{3/4} + \gamma_1 (i\omega)^{5/4} |\omega|\right](s - T)\Big\}\, ds\Bigg].$$

The Fourier transform was inverted numerically for the deformation $f = \exp(-X^2)$. The dependence B(X, T) is shown in Fig. 1 for $\sigma = -2$ at the times T = −2, 0, 6 (curves 1-3). If the solution is close to the limit $B(X, \pm\infty) = -f(X)$ for T = −2 and 6, then at T = 0 it has the nature of large amplitude vibrations where the longitudinal scale of the perturbed flow domain turns out to be much larger than the length of the deformed section of the surface. Such a unique boundary layer excitation is associated with the presence of counterflows in the unperturbed flow as is confirmed by the behavior of the function B(X, T) at the time T = 0 for $\sigma = 2$ (dashed line in Fig. 1).

An exhaustive description of the effect of perturbation of the local solution is obtained successfully in the case of an unperturbed flow with strong counterflows to which the limit form of (2.2) corresponds for $\sigma \to -\infty$. Taking into account that the perimeter ω varies in an unlimited interval, it is expedient to seek the characteristic limits of the solution when obtaining testimates of the integral (2.2). The first occurs when all the components in the exponent of the exponential in (2.2) is of identical order of magnitude, i.e., for $\tau = T\varepsilon^{1/4} \sim 1$, $\zeta = \omega\varepsilon^{1/3} \sim 1$, $\varepsilon = (-\sigma)^{-2} \to 0$. Going over to new variables, we find

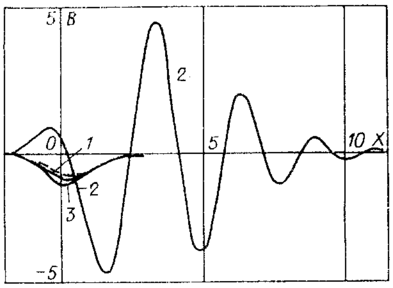

Fig. 1

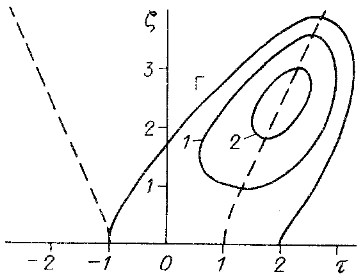

Fig. 2

$$B^* = -f^*(\omega)[1 - \gamma_1 (i\zeta)^{5/4}|\zeta|\Phi(\tau,\ \zeta;\ \varepsilon)],$$
$$\Phi = \frac{1}{\varepsilon}\exp\left[-\frac{1}{\varepsilon}g(\tau,\zeta)\right]\int\limits_{-\infty}^{\tau}\exp\left[\frac{1}{\varepsilon}g(s,\zeta)\right]ds,$$
$$g(s,\zeta) = \frac{\gamma_0}{3}(i\zeta)^{3/4}s^3 - \left[\gamma_0(i\zeta)^{3/4} - \gamma_1(i\zeta)^{5/4}|\zeta|\right]s. \tag{2.3}$$

The solution is represented in a form convenient for estimation of the function Φ by using the saddle-point method for $(\tau,\ \zeta) \sim 1$, $\varepsilon \to 0$. Omitting the details of the analysis, we indicate the final result by limiting ourselves (without loss of generality) to the range of values $\zeta > 0$.

It turns out that a closed curve Γ, symmetric relative to the axis $\zeta = 0$ exists in the plane $(\tau,\ \zeta)$ (the upper half of the curve is shown in Fig. 2). If the point $(\tau,\ \zeta)$ is in the outer domain relative to Γ, then the solution has a quasistationary asymptotic that can be obtained from (2.1) by neglecting the time derivative

$$\Phi(\tau,\ \zeta;\ \varepsilon) = [\gamma_0(i\zeta)^{3/4}(\tau^2 - 1) + \gamma_1(i\zeta)^{5/4}|\zeta|]^{-1} + \dots \tag{2.4}$$

The asymptotic representation of the solution is more complex in the domain of parameters enclosed within Γ

$$\Phi = \left(\frac{\pi}{\varepsilon\gamma_0}\right)^{1/2}(i\zeta)^{-3/8}(-\tau_0)^{-1/4}\exp\left[-\frac{\gamma_0}{3\varepsilon}\Omega(\tau,\zeta)\right] + \dots,$$
$$\tau_0 = -\left(1 - \frac{\gamma_1}{\gamma_0}(i\zeta)^{1/2}|\zeta|\right)^{1/2},\ \tau_{0r} < 0,\ \Omega(\tau,\zeta) = (i\zeta)^{3/4}(\tau - \tau_0)^2(\tau + 2\tau_0). \tag{2.5}$$

Here τ_{0r} is the real part of τ_0. The level lines $\Omega_r = -3$ and -6 (curves 1 and 2) are shown in Fig. 2. On the curve Γ itself $\Omega_r = 0$. The vertex of Γ has the ordinate $\zeta_0 = 3.8226$; ζ is the wave number of a definite harmonic in the spectrum of perturbations caused by the plate deformation. The evolution of harmonics with wave numbers $\zeta > \zeta_0$ in time is described by the quasistationary approximation (2.4) in the whole range of variation of τ. If $0 < \zeta < \zeta_0$ then in a definite time interval (Fig. 2), the smooth quasistationary change in the perturbation is transformed into high-frequency oscillations with exponentially large amplitude (2.5). By virtue of the equality $\zeta = \omega\varepsilon^{1/3}$ the excitation effect is present for harmonics with the wave numbers $|\omega| < \zeta_0\varepsilon^{-1/3}$, $\varepsilon \ll 1$.

Additional analysis shows that excitation is observed only for $|\omega| \gg \varepsilon$. In fact, let $\omega = \varepsilon\mu$, $\varepsilon \to 0$, $(\mu,\ \tau) = O(1)$. We write the representation of the solution in this limit case as

$$B^* = -f^*(\omega)\left\{1 - \varepsilon^2\gamma_1(i\mu)^{5/4}|\mu|\int\limits_{-\infty}^{\tau}\exp[G_1(s,\tau,\mu)]\,ds\right\},$$
$$G_1 = \gamma_0(i\mu)^{3/4}[1/3(s^3 - \tau^3) - (s - \tau)]. \tag{2.6}$$

It can be shown that if $\mu \to +\infty$, then the solution (2.6) goes continuously over into (2.3)-(2.5) as $\zeta \to 0$. It is easy also to establish that all possible modes of the solution (2.2) as $\sigma \to -\infty$ are exhausted by the two mentioned characteristic limits.

There now remains to perform the inverse Fourier transform. Let us present an example of such calculations for deformation with a bounded spectrum

$$f(X) = \sin(\omega_0 X)/(\omega_0 X), \ \omega_0 = \mathrm{const},$$
$$f^* = \pi/\omega_0 \ (|\omega| < \omega_0), \ f^* = 0 \ (|\omega| > \omega_0).$$

As $\varepsilon \to 0$.the expression for the function B in the time interval $-1 < \tau < 2$ has the form

$$B = \varepsilon^{-5/8} \frac{\gamma_1}{2} \left(\frac{\pi}{\gamma_0}\right)^{1/2} \left[\frac{4\omega_0 (i\omega_0)^{7/8} \exp \Theta}{\gamma_0 (\tau+1)^2 (2-\tau)(i\omega_0)^{3/4} + 4i\xi\omega_0} + \text{c. c} \right] + \ldots,$$
$$\Theta = \varepsilon^{-3/4} \left[i\omega_0 \xi + \frac{\gamma_0}{3} (\tau+1)^2 (2-\tau)(i\omega_0)^{3/4} \right].$$

Here $\xi = X\varepsilon^{3/4} = O(1)$, the notation c.c. denotes the complex conjugate expression. Therefore, the perturbation around the selected obstacle has the form of a modulated wave with the characteristic longitudinal dimension $X = O(\varepsilon^{-3/4})$ and wave length $X = O(1)$ in the excited state. The longitudinal scale of the perturbation here turns out to be much greater than the length of the deformation. The mentioned properties of the solution were demonstrated earlier in Fig. 1. Let us also note the nonsymmetry of the solution with respect to time that is associated with the inertial properties of the perturbations.

We examine the physical mechanism of boundary layer excitation in greater detail. We first consider the events occurring for minimal values of the wave numbers from the excitation range. We set $\omega = \varepsilon\mu$ and perform a repeated passage to the limit in (2.1): first $\varepsilon \to 0$, $\mu \sim 1$, then $\mu \to \infty$. We assume here that the quasistationary solution is valid at any time τ. Consequently

$$B^* = -f^*(\varepsilon\mu) \left\{ 1 - \varepsilon^2 \left[\frac{\gamma_1 (i\mu)^{1/2} |\mu|}{\tau^2 - 1} + \ldots \right] + o(\varepsilon^2) \right\}. \tag{2.7}$$

Let us recall that the coefficient γ_1 in (2.1) is in front of the component characterizing the boundary layer interaction with the external flow [1]. Then the first term in (2.7) corresponds to the fact that the boundary layer flows around an obstacle by outlining its shape exactly. The shift of the streamline caused by such a flow results in generation of additional pressure in the external stream. The second term of the expansion (2.7) is due to boundary layer reaction to the pressure perturbation.

The solution (2.7) has a singularity for $|\tau| = 1$. It can be assumed that the singularities are eliminated in small neighborhoods of the points mentioned under the effect of nonstationarity and, possibly, interaction. As $\tau = -1$ is approached from different sides, the second term in (2.7) tends to infinites of different signs. If, for example, the stream is accelerated as $\tau \to -1 - 0$, then during a short time interval near $\tau = -1$ the fluid should be retarded and, moreover, acquire a significant negative velocity (only in this case would the condition of joining with the solution as $\tau \to -1 + 0$ be satisfied successfully). Let us show that this does not occur. Let us examine the neighborhood of the time $\tau = -1$ and from considerations of generality let us select the value of μ such that the nonstationarity and the interaction would influence the motion simultaneously. Then setting $\tau = -1 + \varepsilon^{2/5}\tau_1$, $\mu = \varepsilon^{-16/15}\mu_1$ in (2.1), we obtain in the principal approximation

$$\frac{\partial B^*}{\partial \tau_1} - \left[2\gamma_0 (i\mu_1)^{3/4} \tau_1 - \gamma_1 (i\mu_1)^{5/4} |\mu_1| \right] B^* = 2\gamma_0 (i\mu_1)^{3/4} \tau_1 f^*. \tag{2.8}$$

The solution of this equation that satisfies the condition of joining the solution (2.7) as $\tau_1 \to -\infty$ grows exponentially if $\tau_1 \to +\infty$. This indeed denotes the beginning of boundary layer excitation. Therefore, after the short-range action of nonstationarity the stream goes over into the excited state instead of again acquiring a quasistationary form. It is conceivable that this is possible if and only if the unperturbed flow becomes unstable for $\tau > -1$. Flow stability is characterized by the solution of the homogeneous equation (2.1) which can be written in the form

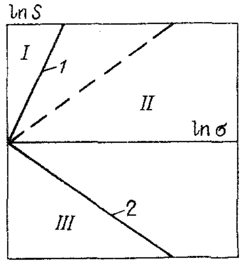

Fig. 3

$$B^* = \exp\left\{-\frac{\gamma_0}{3}(i\omega)^{3/4}T^3 - \left[\gamma_0(i\omega)^{3/4}\sigma + \gamma_1(i\omega)^{5/4}|\omega|\right]T\right\}. \quad (2.9)$$

If the wave number satisfies the condition $\omega = \varepsilon\mu$, $\mu \gg 1$, then instantaneous instability of the solution can be considered. Let us still consider $\omega \ll \varepsilon^{-1/3}$ and let us introduce the fast variable $\tau_2 = \mu^{3/4}\tau$. In the neighborhood of any time τ the solution (2.9), to the accuracy of a factor dependent on just τ but not on τ_2, is represented in the form $B^* = \exp[\gamma_0 i^{3/4}(1 - \tau^2)\tau_2 + O(\mu^{-3/4}\tau_2{}^2)]$. It hence follows directly that in the interval $|\tau| < 1$ the flow is unstable relative to harmonics with the wave numbers $\varepsilon \ll \omega \ll \varepsilon^{-1/3}$.

Equation (2.8) describes flow with interaction. If $\mu_1 \to 0$ then interaction attenuates but the exponential growth of the solution is conserved. Magnification of the interaction ($\mu_1 \to +\infty$) also does not eliminate excitation, however, the beginning of attenuation is delayed here. This effect is seen especially well in the shortwave modes $\omega = \varepsilon^{-1/3}\zeta$, $\zeta \sim 1$. The dashed lines in Fig. 2 superpose the boundaries of the instantaneous instability domain. They are obtained by using the procedure elucidated above applied in the range of the wave numbers $\zeta \sim 1$. It turns out that the flow is unstable if $|\zeta| > |\gamma_0/\gamma_1(\tau^2 - 1)]^{2/3}$. It is interesting that interaction plays a dual role: on one hand expands the instability domain (and increases the instantaneous increments of perturbation growth) and on the other, shifts the time of the beginning of excitation. The shape of the curve Γ in Fig. 2 is indeed determined by the opposition of these two factors.

Let us examine the question for a given class of flows in principle. Let us assume that the deformation mode has a spectrum located in the domain $\zeta \sim 1$ and cut off from zero ($f^* = 0$ if $|\zeta| < \delta$ and δ is a finite number). Then starting from (2.1) we construct the solution in the form of a formal asymptotic series in powers of ε, whose first term is

$$B^* = -\frac{\gamma_0(i\zeta)^{3/4}(\tau^2 - 1)f^*(\varepsilon^{-1/3}\zeta)}{\gamma_0(i\zeta)^{3/4}(\tau^2 - 1) + \gamma_1(i\zeta)^{5/4}|\zeta|}[1 + O(\varepsilon)]. \quad (2.10)$$

It can be seen that such a quasistationary solution is uniformly suitable in an infinite interval of variation of τ and satisfies the conditions for junction with the external solution for $|\tau| \gg 1$. Therefore, the approximation (2.10) satisfies the principle of self-consistency which is considered as the criterion for correctness of the solution of the problem in the method of joined asymptotic expansions. Meanwhile passage to the limit in the exact solution shows that (2.10) is not suitable in a certain bounded interval of variation of τ. In this case the Stokes phenomenon known in the asymptotic theory of differential equations is observed. Indeed, if the time τ is considered a complex variable, then the approximation (2.10) will have two poles displaced from the real axis, whose influence (more accurately, of one of them) also results in a loss of suitability of the quasistationary solution.

3. NONLINEAR INTERACTION MODES IN A FLOW WITHOUT COUNTERCURRENTS

Next we examine the limit forms of the problem (1.1) as $\sigma \to +\infty$. We assume that the deformation has the characteristic longitudinal dimension S such that $f(X) = F(X/S)$ and we execute a change of variable $X = SX_0$, $B(X, T) = -F(X_0) + C(X_0, T)$, $\gamma_2 = 2^{-5/4}\gamma_0{}^{-1}$, $\gamma_3 = \pi^{-1}\gamma^{-1}2^{-1/2}$. We write the problem (1.1) in the form

$$H_0C^2 + (T^2 + \sigma)C = S^{-3/2}J_1(C, X_0, T) - S^{3/4}J_2(C, X_0, T) + S^{-2/3}G(X_0),$$

$$C(X_0, -\infty) = C(-\infty, T) = 0,$$

$$J_1(C, X_0, T) = \gamma_2 \int_{X_0}^{\infty} \frac{\partial^2 C(s, T)}{\partial s^2} \frac{ds}{(s - X_0)^{1/2}},$$

$$J_2(C, X_0, T) = \gamma_3 \int_{-\infty}^{X_0} \frac{\partial C(s, T)}{\partial T} \frac{ds}{(X_0 - s)^{1/4}}, \qquad (3.1)$$

$$G(X_0) = -\gamma_2 \int_{X_0}^{\infty} \frac{d^2 F(s)}{ds^2} \frac{ds}{(s - X_0)^{1/2}}.$$

Passing to the limit $\sigma \to +\infty$ we select the deformation amplitude H_0 so as to conserve the nonlinear nature of the perturbed flow. The problem (3.1) has two characteristic limits corresponding to the separate influence of the nonstationary and interaction effects. In the first case $S \sim \sigma^2$, $T \sim \sigma^{1/2}$, $H_0 \sim \sigma^5$, $C \sim \sigma^{-4}$ (ray 1 in Fig. 3). In the limit the integral J_1 characterizing the interaction effect drops out of (3.1), i.e., this flow mode is a particular case of nonstationary fluid motion in a boundary layer with a given pressure distribution.

The second characteristic limit of the problem is nonlinear quasistationary flow with interaction and is realized for $S \sim \sigma^{-2/3}$, $T \sim \sigma^{1/2}$, $H_0 \sim \sigma$, $C \sim 1$ (ray 2 in Fig. 3). The integral J_2 in (3.1) is here small as compared with the remaining components. The flow modes corresponding to other relationships between the quantities S and σ, $\sigma \gg 1$ (sectors I-III) can be considered as particular cases of the two above-mentioned characteristic limits.

Let us examine the singularities of the flow mode in sector III included between rays 1 and 2 in greater detail. Both integrals J_1 and J_2 in (3.1) are here negligibly small and the solution is represented in the form

$$T = \sigma^{1/2} T_1, \; H_0 = \sigma^2 S^{3/2} H_5, \; C = \sigma^{-1} S^{-3/2} C_5(X_0, T_1) + \ldots,$$
$$2H_5 C_5 = -(1 + T_1^2) + [4H_5 G(X_0) + (1 + T_1^2)^2]^{1/2}.$$

The condition for existence of a real solution at any time has the form $4H_5 G(X_0) \geq -1$, $(|X_0| < \infty)$ and determine (under the condition of uniqueness of the extremum of the function G at the point $X_0 = X_c$) the threshold amplitude of the deformation $H_5 = H_c$ such that $4H_c G_c = -1$, $G_c = G(X_c)$. If $\Delta H = H_5 - H_c \to 0$, $X_0 \to X_c$, $T_1 \to 0$, $G_c'' = G''(X_c)$ then the solution approaches the solution with a continuable singularity

$$2H_5 C_5 = -1 + \left[-\frac{\Delta H}{H_c} + 2H_c G_c'' (X_0 - X_c)^2 + 2T_1^2\right]^{1/2} + \ldots \qquad (3.2)$$

As the deformation amplitude tends asymptotically to the threshold value, the singularity in (3.2) can restore the lost interaction [3, 4] and nonstationarity [5-7] effects that will act in a small time interval. From generality considerations we require that these factors determine the fluid motion in equal degree. Then setting

$$S = \sigma^{2/3} S_6, \; T = \sigma^{-3/10} \beta T_6,$$
$$X_0 = X_c + \sigma^{-4/5} 2\beta \left| G_c / G_c'' \right|^{1/2} X_6,$$
$$H_0 = \sigma^3 H_{60} - \sigma^{7/5} 2 H_{60} \beta^2 H_6,$$
$$C = -(2\sigma^2 H_{60})^{-1} + \sigma^{-\text{[illegible]}/5} 2^{-1/2} \beta H_{60}^{-1} C_6(X_6, T_6) + \ldots,$$
$$H_{60} = -(4G_c)^{-1} S_6^{3/2}, \quad \gamma_6 = 2^{3/4} \gamma_3 \gamma_2^{1/2} \beta^{-5/2},$$
$$\beta = 2^{-2/5} \gamma_2^{2/5} S_6^{-3/5} \left| G_c / G_c'' \right|^{-3/10}$$

and performing the passage to the limit $\sigma \to +\infty$ in (3.1) (the dashed line in Fig. 3), we arrive at the problem

$$C_6^2 - X_6^2 - T_6^2 - H_6 = \int_{X_6}^{+\infty} \frac{\partial^2 C_6(s, T_6)}{\partial s^2} \frac{ds}{(s - X_6)^{1/2}} - \gamma_6 \int_{-\infty}^{X_6} \frac{\partial C_6(s, T_6)}{\partial T_6} \frac{ds}{(X_6 - s)^{1/4}}, \tag{3.3}$$

$C_6 = (X_6^2 + T_6^2 + H_6)^{1/2} + \ldots$ ($T_6 \to -\infty$, $X_6 \to \pm\infty$). The relationship between the nonstationarity and interaction effects is regulated by the parameter γ_6.

The problem (3.3) agrees exactly with the problem of interaction at the leading edge of a thin profile for which the angle of attack changes with time according to a parabolic law by reaching the maximal value at the time $T_6 = 0$ [5-7]. The problem requires numerical solution which is made complicated by the limit case of subcritical angles of attack $H_6 \gg 1$, where the solution can be represented in the form

$$(\xi, \eta) = H_6^{-1/2}(X_6, T_6) = O(1),$$
$$C_6 = H_6^{-1/2} D_1(\xi, \eta) + H_6^{-9/8} D_2(\xi, \eta) + \ldots, \quad D_1 = (1 + \xi^2 + \eta^2)^{1/2},$$
$$D_2 = -\frac{\gamma_6 \eta}{2(1 + \xi^2 + \eta^2)^{1/2}} \int_{-\infty}^{\xi} \frac{ds}{(1 + s^2 + \eta^2)^{1/2}(\xi - s)^{1/4}}.$$

Let us note that in the principal approximation the solution is symmetric in time with respect to the time $\eta = 0$. However, a linear correction to the friction turns out to be antisymmetric and, what is essential, negative for $\eta > 0$. Thus follows the existence of weak hysteresis in the solution for subcritical values of the profile angle of attack. Moreover, the sign of the function D_2 indicates that the stage of diminution of the angle of attack turns out to be more sensitive to separation.

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